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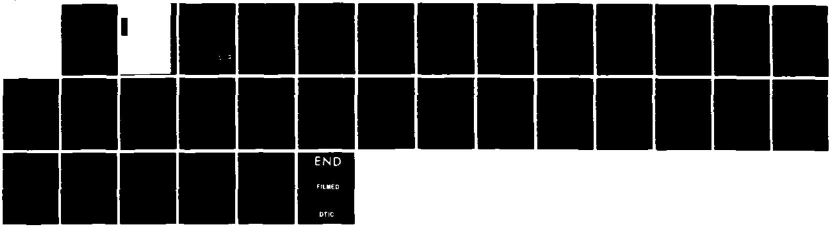

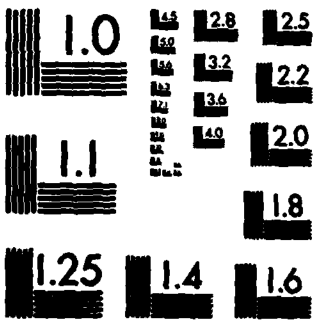

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ELECTROMAGNETIC TRANSMISSION THROUGH A SLOT IN A PERFECTLY CONDUCTING PLANE

Interim Technical Report

by

Joseph R. Mautz
Roger F. Harrington

December 1984

Department of
Electrical and Computer Engineering
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Syracuse, New York 13210

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When a plane wave is incident on a slot in a perfectly conducting ground plane of infinite extent, three quantities of interest are the tangential electric field in the aperture, the transmission coefficient, and the scattering cross section. It is assumed that the tangential electric field in the aperture is transverse to the slot axis and depends only on the coordinate along the slot axis. A computer program is presented to calculate the tangential electric field in the aperture, the transmission coefficient, and the

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20. ABSTRACT (continued)

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scattering cross section. This program is described and listed. Sample input and output data are included for the convenience of the user.

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CONTENTS

ELECTROMAGNETIC TRANSMISSION THROUGH A SLOT IN A PERFECTLY CONDUCTING PLANE

I. INTRODUCTION

Although the computer program in [1] can treat the case where the rectangular aperture is one subsection wide, i.e., $L_y = 1$ in Fig. 1 of [1], this computer program is long and complicated. A computer program was written specifically for the $L_y = 1$ case. Relatively short, this program is described and listed here. It consists of a main program and the subroutines YMAT, PLANE, DECOMP, and SOLVE.

The formulas that are programmed are presented in Section II. The main program is described and listed in Section III, the subroutine YMAT in Section IV, the subroutine PLANE in Section V, and the subroutines DECOMP and SOLVE in Section VI.

II. FORMULATION

The magnetic current $\underline{M}$ on the z < 0 side of the aperture (see [2, Fig. 2]) is expressed as

$$\underline{M} = \sum_{j=1}^{L_x-1} V_j \, \underline{M}_j^x \tag{1}$$

where $\{V_j\}$ are unknown coefficients and $\{\underline{M}_j^x\}$ are expansion functions defined by [1, Eq. (10)]

$$\underline{M}_j^x = \underline{u}_x \, T_j(x) \, P(y) \; , \quad j=1,2,\ldots L_x-1 \tag{2}$$

Here, $\underline{u}_x$ is the unit vector in the x-direction, $T_j(x)$ is the triangle function defined by

$$T_j(x) = \begin{cases} \dfrac{x - (j-1)\Delta x}{\Delta x} & (j-1)\Delta x \le x \le j\Delta x \\ \dfrac{(j+1)\Delta x - x}{\Delta x} & j\Delta x \le x \le (j+1)\Delta x \\ 0 & |x - j\Delta x| \ge \Delta x \end{cases} \tag{3}$$

and P(y) is the pulse function defined by

$$P(y) = \begin{cases} 1 & 0 \le y < \Delta y \\ 0 & \text{all other } y \end{cases} \tag{4}$$

Here, Δx and Δy are, respectively, the aperture subsection lengths in the x and y directions.

The magnetic field incident on the aperture-perforated conducting plane of [1, Fig. 1] is either $\underline{H}_{\theta y}$ or $\underline{H}_{xx}$ where

$$\underline{H}_{\theta y} = \underline{u}_\theta(\theta^{inc})\, e^{jk(x\cos\theta^{inc} + z\sin\theta^{inc})}, \quad \pi \le \theta^{inc} < 2\pi \tag{5}$$

$$\underline{H}_{xx} = \underline{u}_x\, e^{jk(y\cos\phi^{inc} + z\sin\phi^{inc})}, \quad \pi \le \phi^{inc} < 2\pi \tag{6}$$

Here, k is the wave number. In (5), the incident wave comes from the direction for which $y=0$ and $\theta=\theta^{inc}$. In (6), the incident wave comes from the direction for which $x=0$ and $\phi=\phi^{inc}$. In (5), $\underline{u}_\theta(\theta^{inc})$ is the unit vector in the θ direction evaluated at $\theta=\theta^{inc}$. In (5) and (6), the first subscript on $\underline{H}$ denotes the polarization of the magnetic field, and the second subscript denotes the plane of incidence. In (5), the plane of incidence is the $y=0$ plane. In (6), the plane of incidence is the $x=0$ plane. Called the incident magnetic field, the magnetic field (5) or (6) is the field that would exist in free-space, i.e., in the absence of the aperture-perforated conducting plane.

When the incident magnetic field is given by (5), the coefficients $\{V_j\}$ in (1) are the elements of the column vector $\vec{V}$ that satisfies

$$Y\vec{V} = -(\vec{P}^{inc})_{\theta y} \tag{7}$$

where the ith element of the column vector $(\vec{P}^{inc})_{\theta y}$ is called $(P_i^{inc})_{\theta y}$ and is given by [1, Eq. (59)]

$$(P_i^{inc})_{\theta y} = 2\Delta x\Delta y \sin\theta^{inc} \left(\frac{\sin\frac{k\Delta x\cos\theta^{inc}}{2}}{\frac{k\Delta x\cos\theta^{inc}}{2}}\right)^2 e^{jki\Delta x\cos\theta^{inc}}, \qquad i=1,2,\ldots L_x-1 \tag{8}$$

In (7), Y is the square matrix whose ij element is called Y_{ij} and is given by [1, Eq. (23)]

$$\begin{aligned} Y_{ij} = \frac{j\Delta x\Delta y}{\pi\eta} \Big[&\frac{1}{2} I_c(j-i) - \frac{1}{2} I_x(j-i+1) + \frac{(j-i+3/2)}{2} I_c(j-i+1) \\ &+ \frac{1}{2} I_x(j-i-1) - \frac{(j-i-3/2)}{2} I_c(j-i-1) + \frac{1}{(k\Delta x)^2} (I_c(j-i+1) \\ &- 2I_c(j-i) + I_c(j-i-1))\Big] \end{aligned} \tag{9}$$

The j that appears in the factor $j\Delta x\Delta y/(\pi\eta)$ on the right-hand side of (9) is $\sqrt{-1}$. Each of the rest of the j's on the right-hand side of (9) is the subscript j on Y_{ij}. In (9), η is the impedance of free space. Moreover, I_c and I_x are given by [1, Eqs. (27) and (28)]

$$I_c(i) = 2\int_0^{y_u} dy \int_{x_\ell}^{x_u} dx \frac{e^{-j\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} \tag{10}$$

$$I_x(i) = \frac{2}{k\Delta x}\int_0^{y_u} dy \int_{x_\ell}^{x_u} dx \, \frac{x \, e^{-j\sqrt{x^2+y^2}}}{\sqrt{x^2 + y^2}} \tag{11}$$

where

$$y_u = k\Delta y/2 \tag{12}$$

$$x_u = (i + 1/2)k\Delta x \tag{13}$$

$$x_\ell = (i - 1/2)k\Delta x \tag{14}$$

It is evident that $I_c(i)$ is even in i and that $I_x(i)$ is odd in i.

When the incident magnetic field is given by (6), the coefficients $\{V_j\}$ in (1) are the elements of the column vector $\vec{V}$ that satisfies

$$Y\vec{V} = -(\vec{P}^{inc})_{xx} \tag{15}$$

where the ith element of the column vector $(\vec{P}^{inc})_{xx}$ is called $(P_i^{inc})_{xx}$ and is given by [1, Eq. (65)]

$$(P_i^{inc})_{xx} = -2\Delta x\Delta y \left(\frac{\sin \frac{k\Delta y \cos \phi^{inc}}{2}}{\frac{k\Delta y \cos \phi^{inc}}{2}}\right) e^{j(k\Delta y/2)\cos \phi^{inc}}, \quad i=1,2,\ldots L_x-1 \tag{16}$$

In (9),

$$2-L_x \le j-i \le L_x-2 \tag{17}$$

but, because (9) is even in j-i,

$$0 \le j-i \le L_x-2 \tag{18}$$

is sufficient. Therefore,

$$-1 \le i \le L_x-1 \tag{19}$$

is sufficient in (10) and (11).

The integrals (10) and (11) are evaluated by using the following four term approximation [1, Eq. (39)]

$$e^{-jr} \simeq e^{-jr_i}[1-j(r-r_i) - \frac{1}{2}(r-r_i)^2 + \frac{j}{6}(r-r_i)^3] \tag{20}$$

where

$$r = \sqrt{x^2 + y^2} \tag{21}$$

$$r_i = |ik\Delta x| \tag{22}$$

Substitution of (20) into (10) gives [1, Eq. (42)]

$$I_c(i) = (C_1U_1 + C_2U_2 + C_3U_3 + C_4U_4)2e^{-jr_i} \tag{23}$$

where

$$U_1 = 1 - \frac{r_i^2}{2} + jr_i\ (1 - \frac{r_i^2}{6}) \tag{24}$$

$$U_2 = r_i - j\ (1 - \frac{r_i^2}{2}) \tag{25}$$

$$U_3 = -\frac{1}{2}(1 + jr_i) \tag{26}$$

$$U_4 = \frac{j}{6} \tag{27}$$

$$C_n = \int_0^{y_u} dy \int_{x_\ell}^{x_u} dx\ r^{n-2}\ , \quad n = 1,2,3,4 \tag{28}$$

Substitution of (20) into (11) gives

$$I_x(i) = (X_1U_1 + X_2U_2 + X_3U_3 + X_4U_4)\ \frac{2e^{-jr_i}}{k\Delta x} \tag{29}$$

where U_1, U_2, U_3, and U_4 are given by (24)-(27) and

$$X_n = \int_0^{y_u} dy \int_{x_\ell}^{x_u} dx \; x r^{n-2} \quad , \quad n=1,2,3,4 \tag{30}$$

Using the indefinite integrals [1, Eqs. (46)-(54)], we obtain

$$C_1 = A_{xu} - A_{x\ell} + A_{yu} \tag{31}$$

$$C_2 = y_u k\Delta x \tag{32}$$

$$C_3 = \frac{y_u}{3} (x_u r_4 - x_\ell r_3) + \frac{1}{6} (x_u^2 A_{xu} - x_\ell^2 A_{x\ell} + y_u^2 A_{yu}) \tag{33}$$

$$C_4 = \frac{1}{3} y_u \; (x_u r_4^2 - x_\ell r_3^2) \tag{34}$$

$$X_1 = \frac{1}{2} (y_u (r_4 - r_3) + x_u A_{xu} - x_\ell A_{x\ell}) \tag{35}$$

$$X_2 = r_i C_2 \text{ sign } (i) \tag{36}$$

$$X_3 = y_u \left(\frac{r_4^3 - r_3^3}{12} + \frac{x_u^2 r_4 - x_\ell^2 r_3}{8}\right) + \frac{1}{8} (x_u^3 A_{xu} - x_\ell^3 A_{x\ell}) \tag{37}$$

$$X_4 = r_i y_u k\Delta x \left(r_i^2 + \frac{(k\Delta x)^2}{4} + \frac{y_u^2}{3}\right) \text{ sign } (i) \tag{38}$$

where sign (i) denotes the algebraic sign of i. If i=0, then sign (i) is inconsequential because the term that multiplies it is zero. In (31)-(38),

$$r_3 = \sqrt{x_\ell^2 + y_u^2} \tag{39}$$

$$r_4 = \sqrt{x_u^2 + y_u^2} \tag{40}$$

$$A_{xu} = x_u \log \left(\frac{y_u + r_4}{|x_u|}\right) \tag{41}$$

$$A_{x\ell} = x_\ell \log \left(\frac{y_u + r_3}{|x_\ell|}\right) \tag{42}$$

$$A_{yu} = y_u \log \left(\frac{x_u + r_4}{x_\ell + r_3}\right) \tag{43}$$

Here, log denotes the natural logarithm.

Substituting Y/2 for Y^b in [2, Eq. (27)], we obtain for the complex power P_t transmitted through the aperture

$$P_t = \frac{1}{2} \tilde{V}[Y\vec{V}]^* \tag{44}$$

where $\tilde{V}$ is the transpose of $\vec{V}$ and * denotes the complex conjugate. Since the incident magnetic field is given by either (5) or (6), $Y\vec{V}$ is given by either (7) or (15) so that (44) reduces to

$$P_t = -\frac{1}{2} \tilde{V}\vec{P}^* \tag{45}$$

where $\vec{P}$ is $(\vec{P}^{inc})_{\theta y}$ if the incident magnetic field is given by (5) and $\vec{P}$ is $(\vec{P}^{inc})_{xx}$ if the incident magnetic field is given by (6).

The transmission coefficient T is the ratio of the real power transmitted through the aperture to the real power P_{inc} incident on the aperture. Since the incident magnetic field is given by either (5) or (6), we obtain

$$P_{inc} = \eta L_x \Delta x \Delta y \cos \alpha \tag{46}$$

where α is the angle between the propagation vector of the incident wave and $\underline{u}_z$. Here, $\underline{u}_z$ is the unit vector in the z direction. Since the incident magnetic field is given by (5) or (6), inspection of [1, Fig. 1] reveals that

$$\cos \alpha = -\sin \beta \tag{47}$$

where β is either the angle θ^{inc} in (5) or the angle ϕ^{inc} in (6). From (45)-(47), we obtain

$$T = \frac{\mathrm{Real}(\tilde{V}\vec{P}^*)}{2\eta L_x \Delta x \Delta y \sin\beta} \tag{48}$$

The scattering cross section $\tau(\theta,\phi)$ is the area that the incident power per unit area must be multiplied by in order to obtain the power which, when radiated omnidirectionally into the $z > 0$ half space, would produce the actual power per unit area at (θ,ϕ). The above definition of τ leads to [1, Eq. (72)]

$$\tau/\lambda^2 = \frac{k^4}{32\pi^3\eta^2} |\tilde{P}^m\vec{V}|^2 \tag{49}$$

where λ is the wavelength and $\tilde{P}^m$ is the transpose of a measurement vector $\vec{P}^m$. For the θ polarized pattern in the $y=0$ plane, τ is called $(\tau)_{\theta y}$ and $\vec{P}^m$ is $(\vec{P}^m)_{\theta y}$ where $(\vec{P}^m)_{\theta y}$ is the column vector whose ith element is given by (8) with the angle of incidence θ^{inc} replaced by the observation or "measurement" angle θ^m.

$$(\tau)_{\theta y}/\lambda^2 = \frac{k^4}{32\pi^3\eta^2} |(\tilde{P}^m)_{\theta y}\vec{V}|^2 \tag{50}$$

For the x polarized pattern in the $x=0$ plane, τ is called $(\tau)_{xx}$ and $\vec{P}^m$ is $(\vec{P}^m)_{xx}$ where $(\vec{P}^m)_{xx}$ is the column vector whose ith element is given by (16) with the angle of incidence ϕ^{inc} replaced by the measurement angle ϕ^m.

$$(\tau)_{xx}/\lambda^2 = \frac{k^4}{32\pi^3\eta^2} |(\tilde{P}^m)_{xx}\vec{V}|^2 \tag{51}$$

III. THE MAIN PROGRAM

The main program uses the subroutines YMAT, PLANE, DECOMP, and SOLVE to calculate the coefficients $\{V_j\}$ appearing in expression (1) for the magnetic current, the transmission coefficient (48), and the scattering cross sections per square wavelength (50) and (51). The main program is described and listed in this section. Sample input and output data are provided so that the user can verify that the program is running properly.

In the listing of the main program, line 4 defines the input data file and line 5 defines the output data file. The input data are read according to lines 10 and 11 which are

```
      READ(20,11) LX, LI, NTH, DX, DY, TH
   11 FORMAT(3I3, 3E14.7)
```

Here, LX is L_x, DX is $\Delta x/\lambda$, and DY is $\Delta y/\lambda$ where λ is the wavelength, and, as in Section II, $L_x \Delta x$ is the length of the aperture in the x direction and Δy is the width of the aperture in the y direction. LX is a positive integer greater than or equal to 2. LI is either 1 or 2. If LI is 1, then the incident magnetic field is given by (5) and TH is θ^{inc} of (5). If LI is 2, then the incident magnetic field is given by (6) and TH is ϕ^{inc} of (6). The input variable TH is in degrees. The normalized cross section (50) is calculated at

$$\theta^m = (J-1)\pi/(NTH-1), \qquad J = 1,2,\ldots NTH \tag{52}$$

and is written on the output data file under the heading TAU1. The normalized cross section (51) is calculated at

$$\phi^m = (J-1)\pi/(NTH-1) \ , \quad J = 1,2,\ldots,NTH \tag{53}$$

and is written on the output data file under the heading TAU2. The right-hand sides of both (52) and (53) are in radians.

Minimum allocations are given by

```
COMPLEX Y(N*N), P(2*N), B(N), V(N)
DIMENSION IPS(N)
```

where

$$N = LX - 1 \tag{54}$$

Line 20 stores by columns in Y the elements of the matrix $\frac{\pi\eta}{j\Delta x\Delta y}$ Y where Y appears in (7). Line 24 stores in P(1) to P(N) the elements of $\frac{1}{2\Delta x\Delta y} (\vec{P}^{inc})_{\theta y}$ where $(\vec{P}^{inc})_{\theta y}$ appears in (7). Here, N is given by (54). Line 24 also stores in P(N+1) to P(2*N) the elements of $\frac{1}{2\Delta x\Delta y} (\vec{P}^{inc})_{xx}$ where $(\vec{P}^{inc})_{xx}$ appears in (15). Equation (7) is recast as

$$[\frac{\pi\eta}{j\Delta x\Delta y} Y]\vec{V} = j2\pi\eta \ [\frac{1}{2\Delta x\Delta y} (\vec{P}^{inc})_{\theta y}] \tag{55}$$

Equation (15) is recast as

$$[\frac{\pi\eta}{j\Delta x\Delta y} Y]\vec{V} = j2\pi\eta \ [\frac{1}{2\pi\Delta x\Delta y} (\vec{P}^{inc})_{xx}] \tag{56}$$

The square matrix $[\frac{\pi\eta}{j\Delta x\Delta y} Y]$ is common to the left-hand sides of both (55) and (56). This is the matrix that resides in the computer program variable Y. The bracketed quantity on the right-hand side of (55) is the column vector that resides in the computer program variables P(1) to P(N) where N is given by (54). The bracketed quantity on the right-hand side of (56) is the column vector that resides in the computer program variables P(N+1) to P(2*N). If LI is 1, DO loop 16 performs the multiplication by the factor UV = $2j\pi\eta$ in (55) in order to store the elements of the right-hand side of (55) in B(1) to B(N). If LI is 2, DO loop 16 stores the elements of the right-hand side of (56) in B(1) to B(N). If LI is 1, lines 33 and 34 put in V(1) to V(N) the elements of the column vector $\vec{V}$ that satisfies (55). If LI is 2, lines 33 and 34 put in V(1) to V(N) the elements of the column vector $\vec{V}$ that satisfies (56). Since the elements of $\vec{V}$ are the $\{V_j\}$ of (1), it is evident that the coefficients $\{V_j\}$ in the expansion (1) for the magnetic current $\underline{M}$ will reside in V(1) to V(N).

Equation (48) is recast as

$$T = \frac{\text{Real } \left(\tilde{V} \frac{1}{2\Delta x\Delta y} \vec{P}^*\right)}{\eta L_x \sin\beta} \tag{57}$$

In (57), β is TH and $\frac{1}{2\Delta x\Delta y}\vec{P}$ is the column vector that stored in either P(1) to P(N) or P(N+1) to P(2*N) according as LI is either 1 or 2. With regard to (57), DO loop 17 accumulates $\tilde{V}\frac{1}{2\Delta x\Delta y}\vec{P}^*$ in U1. Line 44 puts T of (57) in the computer program variable T.

Equation (50) is recast as

$$(\tau)_{\theta y}/\lambda^2 = \frac{(k^2\Delta x\Delta y)^2}{8\pi^3\eta^2}\left|\frac{1}{2\Delta x\Delta y}(\tilde{P}^m)_{\theta y}\vec{V}\right|^2 \tag{58}$$

Equation (51) is recast as

$$(\tau)_{xx}/\lambda^2 = \frac{(k^2\Delta x\Delta y)^2}{8\pi^3\eta^2}\left|\frac{1}{2\Delta x\Delta y}(\tilde{P}^m)_{xx}\vec{V}\right|^2 \tag{59}$$

The index J of DO loop 19 obtains

$$\theta^m = (J-1)\pi/(NTH-1) \tag{60}$$

in (58) and

$$\phi^m = (J-1)\pi/(NTH-1) \tag{61}$$

in (59). The value of the right-hand side of (61) is the same as that of (60). Inside DO loop 19, line 53 puts this common value in TH. With regard to (58) and (59), line 54 puts the elements of $\frac{1}{2\Delta x\Delta y}\ (\vec{P}^m)_{\theta y}$ in P(1) to P(N) and the elements of $\frac{1}{2\Delta x\Delta y}\ (\vec{P}^m)_{xx}$ in P(N+1) to P(2*N). DO loop 21 lies inside DO loop 20 whose index is K. If K = 1, DO loop 21 accumulates $\frac{1}{2\Delta x\Delta y}\ (\tilde{P}^m)_{\theta y}\vec{V}$ in U1. If K = 2, DO loop 21 accumulates $\frac{1}{2\Delta x\Delta y}\ (\tilde{P}^m)_{xx}\vec{V}$ in U1. When K = 1, line 64 puts $(\tau)_{\theta y}/\lambda^2$ of (58) with θ^m given by (60) in TAU(1). When K = 2, line 64 puts $(\tau)_{xx}/\lambda^2$ of (59) with ϕ^m given by (61) in TAU(2).

```
C         LISTING OF THE MAIN PROGRAM
          COMPLEX U,UV,Y(1600),P(100),B(40),V(40),U1,CONJG
          DIMENSION IPS(40),TAU(2)
          OPEN(UNIT=20,FILE='MAUTZ3.DAT')
          OPEN(UNIT=21,FILE='MAUTZ4.DAT')
          PI=3.141593
          ETA=376.730
          U=(0.,1.)
          UV=2.*PI*ETA*U
          READ(20,11) LX,LI,NTH,DX,DY,TH
   11     FORMAT(3I3,3E14.7)
          WRITE(21,12) LX,LI,NTH,DX,DY,TH
   12     FORMAT('  LX LI NTH',5X,'DX',12X,'DY',12X,'TH'/1X,
        1 3I3,3E14.7)
          BK=2.*PI
          DX=DX*BK
          DY=DY*BK
          PB=180./PI
          TH=TH/PB
          CALL YMAT(LX,DX,DY,Y)
          WRITE(21,13) (Y(I),I=1,3)
   13     FORMAT(' Y'/(1X,6E11.4))
          N=LX-1
          CALL PLANE(TH,LX,DX,DY,P)
          WRITE(21,14) (P(I),I=1,3)
   14     FORMAT(' P'/1X,6E11.4)
          IA=(LI-1)*N
          IB=IA
          DO 16 J=1,N
          IB=IB+1
          B(J)=UV*P(IB)
   16     CONTINUE
          CALL DECOMP(N,IPS,Y)
          CALL SOLVE(N,IPS,Y,B,V)
          WRITE(21,24) (V(I),I=1,N)
   24     FORMAT(' COEFFICIENTS V OF MAGNETIC CURRENT ',
        1 'EXPANSION FUNCTIONS'/(1X,6E11.4))
          U1=0.
          IB=IA
          DO 17 J=1,N
          IB=IB+1
          U1=U1+V(J)*CONJG(P(IB))
   17     CONTINUE
          T=REAL(U1)/(LX*ETA*SIN(TH))
          WRITE(21,18) T
   18     FORMAT(' TRANSMISSION COEFFICIENT T=',E14.7)
          CT=DX*DY/(PI*ETA)
          CT=CT*CT/(8.*PI)
          DTH=PI/(NTH-1)
          WRITE(21,23)
```

```
23    FORMAT(' ANGLE',4X,'TAU1',7X,'TAU2')
      DO 19 J=1,NTH
      TH=(J-1)*DTH
      CALL PLANE(TH,LX,DX,DY,P)
      TH=TH*P8
      J1=0
      DO 20 K=1,2
      U1=0.
      DO 21 I=1,N
      J1=J1+1
      U1=U1+P(J1)*V(I)
21    CONTINUE
      H=U1*CONJG(U1)
      TAU(K)=CT*H
20    CONTINUE
      WRITE(21,22) TH,(TAU(I),I=1,2)
22    FORMAT(1X,F7.2,2E11.4)
19    CONTINUE
      STOP
      END
```

INPUT DATA IN THE FILE MAUTZ3.DAT

```
 5   1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
```

```
OUTPUT DATA IN THE FILE NAUTZ4.DAT

 LX LI NTH       DX               DY               TH
  5  1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
Y
-0.1531E+02-0.6526E-01 0.6646E+01-0.6463E-01 0.1312E+01-0.6274E-01
P
-0.1000E+01-0.1471E-06-0.1000E+01-0.2941E-06-0.1000E+01-0.4412E-06
COEFFICIENTS V OF MAGNETIC CURRENT EXPANSION FUNCTIONS
 0.4511E+02 0.5916E+03 0.6238E+02 0.8153E+03 0.6238E+02 0.8153E+03
 0.4511E+02 0.5916E+03
TRANSMISSION COEFFICIENT T= 0.1141263E+00
ANGLE     TAU1        TAU2
   0.00 0.0000E+00 0.2186E-02
  10.00 0.5885E-04 0.2186E-02
  20.00 0.2308E-03 0.2188E-02
  30.00 0.5016E-03 0.2190E-02
  40.00 0.8462E-03 0.2193E-02
  50.00 0.1228E-02 0.2196E-02
  60.00 0.1602E-02 0.2199E-02
  70.00 0.1918E-02 0.2202E-02
  80.00 0.2129E-02 0.2203E-02
  90.00 0.2204E-02 0.2204E-02
 100.00 0.2129E-02 0.2203E-02
 110.00 0.1918E-02 0.2202E-02
 120.00 0.1602E-02 0.2199E-02
 130.00 0.1228E-02 0.2196E-02
 140.00 0.8462E-03 0.2193E-02
 150.00 0.5016E-03 0.2190E-02
 160.00 0.2308E-03 0.2188E-02
 170.00 0.5885E-04 0.2186E-02
 180.00 0.2088E-15 0.2186E-02
```

IV. THE SUBROUTINE YMAT

The subroutine YMAT(LX, DX, DY, Y) stores by columns in Y the matrix $\frac{\pi\eta}{j\Delta x\Delta y}$ Y that appears in (55) and (56). The first three arguments of YMAT are input arguments. The aperture is LX subsections long in the x direction and one subsection wide in the y-direction. DX is $k\Delta x$ and DY is $k\Delta y$ where k is the wave number, Δx is the subsection length in the x direction, and Δy is the subsection width in the y direction.

Minimum allocations are given by

COMPLEX TC(LX+1), TX(LX+1), YXX(N), Y(N*N)

where N is given by (54).

Inside DO loop 16, $I_c(I-1)$ of (23) is put in the computer program variable TC(I+1). Here, I is the index of DO loop 16. Also inside DO loop 16, $I_x(I-1)$ of (29) is put in the computer program variable TX(I+1). The logic inside DO loop 16 is best understood by building up a table of variables in YMAT versus expressions in terms of variables in Section II.

Variables in YMAT	Expressions in Section II
I	i+1 where i appears in (23) and (29)
YU	y_u of (12)
XU	x_u of (13)
XL	x_ℓ of (14)
R1	r_1 of (22)
U1	U_1 of (24)
U2	U_2C_2 of (25) and (32)
U3	U_3 of (26)
U4	$\frac{1}{6}$ of (27)

EX	$2e^{-jr_i}$ of (23) and (29)
R3	r_3 of (39)
R4	r_4 of (40)
AXU	A_{xu} of (41)
AXL	$A_{x\ell}$ of (42)
AYU	A_{yu} of (43)
C1	C_1 of (31)
C3	C_3 of (33)
C4	C_4 of (34)
TC(I+1)	$I_c(i)$ of (23) with i = I-1
X1	X_1 of (35)
X3	X_3 of (37)
X4	X_4 of (38)
TX(I+1)	$I_x(i)$ of (29) with i = I-1

Taking advantage of the fact that $I_c(i)$ is even in i, line 46 stores $I_c(-1)$ in TC(1). Taking advantage of the fact that $I_x(i)$ is odd in i, line 47 stores $I_x(-1)$ in TX(1).

Inside DO loop 20, lines 51 and 52 put in YXX(J-1) the square bracketed term on the right-hand side of (9) with $j-i$ = J-2. Now, we have

$$\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij} = \mathrm{YXX}(j-i+1), \quad j-i = 0,1,2,\ldots,L_x-2 \tag{62}$$

Because Y_{ij} is even in $(j-i)$, (62) becomes

$$\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij} = \mathrm{YXX}(|j-i|+1), \quad |j-i| = 0,1,2,\ldots, L_x-2 \tag{63}$$

Inside nested do loops 23 and 21, line 59 puts $\frac{\pi\eta}{j\Delta x\Delta y} Y_{IJ}$ of (63) in the computer program variable Y(I + (J-1)*N) where N is given by (54).

```
C     LISTING OF THE SUBROUTINE YMAT
      SUBROUTINE YMAT(LX,DX,DY,Y)
      COMPLEX U,U1,U2,U3,U4,EX,TC(100),TX(100)
      COMPLEX YXX(100),Y(1600)
      DX2=DX*DX
      N=LX-1
      U=(0.,1.)
      U4=.1666667*U
      YU=.5*DY
      YUD=YU*DX
      YU2=YU*YU
      YU3=.3333333*YU
      YU4=.25*DX2+YU3*YU
      DO 16 I=1,LX
      IP=I+1
      XU=(I-.5)*DX
      XU2=XU*XU
      XL=XU-DX
      XL2=XL*XL
      R1=(I-1)*DX
      R2=R1*R1
      RU1=1.-.5*R2
      U1=RU1+R1*(1.-.1666667*R2)*U
      U2=(R1-RU1*U)*YUD
      U3=-.5-.5*R1*U
      EX=2.*(COS(R1)-U*SIN(R1))
      R7=XL2+YU2
      R8=XU2+YU2
      R3=SQRT(R7)
      R4=SQRT(R8)
      AXU=XU*ALOG((YU+R4)/XU)
      AXL=XL*ALOG((YU+R3)/ABS(XL))
      AYU=YU*ALOG((XU+R4)/(XL+R3))
      C1=AXU-AXL+AYU
      C3=YU3*(XU*R4-XL*R3)+.1666667*(XU2*AXU-XL2*AXL+YU2*AYU)
      C4=YU3*(XU*R8-XL*R7)
      TC(IP)=(C1*U1+U2+C3*U3+C4*U4)*EX
      AXU=XU*AXU
      AXL=XL*AXL
      X1=.5*(YU*(R4-R3)+AXU-AXL)
      X3=YU*(.8333333E-1*(R8*R4-R7*R3)+.125*(XU2*R4-XL2*R3))
     1+.125*(XU2*AXU-XL2*AXL)
      X4=R1*YUD*(R2+YU4)
      TX(IP)=(X1*U1+R1*U2+X3*U3+X4*U4)*EX/DX
   16 CONTINUE
      TC(1)=TC(3)
      TX(1)=-TX(3)
      DO 20 J=2,LX
      JM=J-1
      JP=J+1
      YXX(JM)=.5*(TC(J)-TX(JP)+(J-.5)*TC(JP)+TX(JM)-
     1(J-3.5)*TC(JM))+(TC(JP)-2.*TC(J)+TC(JM))/DX2
   20 CONTINUE
      JY=0
      DO 23 J=1,N
      DO 21 I=1,N
      JY=JY+1
      K=IABS(J-I)+1
      Y(JY)=YXX(K)
   21 CONTINUE
   23 CONTINUE
      RETURN
      END
```

V. THE SUBROUTINE PLANE

The subroutine PLANE(TH, LX, DX, DY, P) stores in P(1) to P(N) the quantities

$$\frac{1}{2\Delta x\Delta y}\,(P_i^{inc})_{\theta y} = \sin\theta^{inc}\left(\frac{\sin\left(\frac{k\Delta x\cos\theta^{inc}}{2}\right)}{\frac{k\Delta x\cos\theta^{inc}}{2}}\right)^2 e^{jki\Delta x\cos\theta^{inc}}\,,\qquad i=1,2,\ldots,N \tag{64}$$

where (64) comes from (8) and N is (L_x-1). The aperture is L_x subsections long in the x direction and one subsection wide in the y direction. In (64), k is the wave number, Δx is the subsection length in the x direction, and Δy is the width of the aperture in the y direction. Moreover, the subroutine PLANE stores in P(N+1) to P(2N) the quantities

$$\frac{1}{2\Delta x\Delta y}\,(P_i^{inc})_{xx} = -\left(\frac{\sin\frac{k\Delta y\cos\phi^{inc}}{2}}{\frac{k\Delta y\cos\phi^{inc}}{2}}\right) e^{j(k\Delta y/2)\cos\phi^{inc}}\,,\qquad i=1,2,\ldots,N \tag{65}$$

where (65) comes from (16). The angle ϕ^{inc} in (65) is assumed to be the same as θ^{inc} in (64). The first four arguments of PLANE are input arguments. In radians, TH is the common value of θ^{inc} and ϕ^{inc} in (64) and (65). Furthermore, LX is L_x, DX is $k\Delta x$, and DY is $k\Delta y$. LX is a positive integer greater than or equal to 2.

The minimum allocation for P is given by

```
COMPLEX P(2*N)
```

where N is given by (54).

Lines 5 and 6 set

$CX = k\Delta x\cos(TH)$

$CY = \frac{1}{2}k\Delta y\cos(TH)$

If cos (TH) $\neq$ 0, then lines 11 to 14 set

$$SX = \sin(TH) \left(\frac{\sin\left(\frac{k\Delta x \cos(TH)}{2}\right)}{\frac{k\Delta x \cos(TH)}{2}} \right)^2$$

$$SY = - \frac{\sin\left(\frac{k\Delta y \cos(TH)}{2}\right)}{\frac{k\Delta y \cos(TH)}{2}}$$

If cos (TH) = 0, then lines 8 and 9 set SX and SY equal to the limits of the above two expressions as cos (TH) goes to zero, namely

$$SX = \sin(TH)$$

$$SY = -1$$

Inside DO loop 13, line 19 puts in P(I) the right-hand side of (64) with i replaced by I where I is the index of DO loop 13. Note that the right-hand side of (65) does not depend on i. Line 21 puts in U1 the right-hand side of (65). DO loop 14 puts the right-hand side of (65) in P(N+1) to P(2*N) where N is given by (54).

```
C       LISTING OF THE SUBROUTINE PLANE
        SUBROUTINE PLANE(TH,LX,DX,DY,P)
        COMPLEX U,U1,P(100)
        CS=COS(TH)
        CX=DX*CS
        CY=.5*DY*CS
        IF(CS) 11,10,11
10      SX=SIN(TH)
        SY=-1.
        GO TO 12
11      SX=.5*CX
        SX=SIN(SX)/SX
        SX=SIN(TH)*SX*SX
        SY=-SIN(CY)/CY
12      U=(0.,1.)
        N=LX-1
        DO 13 I=1,N
        S=I*CX
        P(I)=SX*(COS(S)+U*SIN(S))
13      CONTINUE
        U1=SY*(COS(CY)+U*SIN(CY))
        DO 14 J=1,N
        I=J+N
        P(I)=U1
14      CONTINUE
        RETURN
        END
```

VI. THE SUBROUTINES DECOMP AND SOLVE

The subroutines DECOMP(N, IPS, UL) and SOLVE(N, IPS, UL, B, X) solve a system of N linear equations in N unknowns. The input to DECOMP consists of N and the N by N matrix of coefficients on the left-hand side of the matrix equation stored by columns in UL. The output from DECOMP is IPS and UL. This output is fed into SOLVE. The rest of the input to SOLVE consists of N and the column of coefficients on the right-hand side of the matrix equation stored in B. SOLVE puts the solution to the matrix equation in X.

Minimum allocations are given by

```
COMPLEX UL(N*N)
DIMENSION SCL(N), IPS(N)
```

in DECOMP and by

```
COMPLEX UL(N*N), B(N), X(N)
DIMENSION IPS(N)
```

in SOLVE.

More detail concerning DECOMP and SOLVE is on pages 46-49 of [3].

```
C     LISTING OF THE SUBROUTINE DECOMP
      SUBROUTINE DECOMP(N,IPS,UL)
      COMPLEX UL(1600),PIVOT,EM
      DIMENSION SCL(40),IPS(40)
      DO 5 I=1,N
      IPS(I)=I
      RN=0.
      J1=I
      DO 2 J=1,N
      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
      J1=J1+N
      IF(RN-ULM) 1,2,2
1     RN=ULM
2     CONTINUE
      SCL(I)=1./RN
5     CONTINUE
      NM1=N-1
      K2=0
      DO 17 K=1,NM1
      BIG=0.
      DO 11 I=K,N
      IP=IPS(I)
      IPK=IP+K2
      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
      IF(SIZE-BIG) 11,11,10
10    BIG=SIZE
      IPV=I
11    CONTINUE
      IF(IPV-K) 14,15,14
14    J=IPS(K)
      IPS(K)=IPS(IPV)
      IPS(IPV)=J
15    KPP=IPS(K)+K2
      PIVOT=UL(KPP)
      KP1=K+1
      DO 16 I=KP1,N
      KP=KPP
      IP=IPS(I)+K2
      EM=-UL(IP)/PIVOT
18    UL(IP)=-EM
      DO 16 J=KP1,N
      IP=IP+N
      KP=KP+N
      UL(IP)=UL(IP)+EM*UL(KP)
16    CONTINUE
      K2=K2+N
17    CONTINUE
      RETURN
      END
```

```
C         LISTING OF THE SUBROUTINE SCLVE
          SUBROUTINE SOLVE(N,IPS,UL,B,X)
          COMPLEX UL(1600),B(40),X(40),SUM
          DIMENSION IPS(40)
          NP1=N+1
          IP=IPS(1)
          X(1)=B(IP)
          DO 2 I=2,N
          IP=IPS(I)
          IPB=IP
          IM1=I-1
          SUM=0.
          DO 1 J=1,IM1
          SUM=SUM+UL(IP)*X(J)
 1        IP=IP+N
 2        X(I)=B(IPB)-SUM
          K2=N*(N-1)
          IP=IPS(N)+K2
          X(N)=X(N)/UL(IP)
          DO 4 IBACK=2,N
          I=NP1-IBACK
          K2=K2-N
          IPI=IPS(I)+K2
          IP1=I+1
          SUM=0.
          IP=IPI
          DO 3 J=IP1,N
          IP=IP+N
 3        SUM=SUM+UL(IP)*X(J)
 4        X(I)=(X(I)-SUM)/UL(IPI)
          RETURN
          END
```

REFERENCES

[1] J. R. Mautz and R. F. Harrington, "Electromagnetic Transmission through a Rectangular Aperture in a Perfectly Conducting Plane," Report TR-76-1, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210, February 1976.

[2] R. F. Harrington and J. R. Mautz, "A Generalized Network Formulation for Aperture Problems," Report TR-75-13, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210, November 1975.

[3] J. R. Mautz and R. F. Harrington, "Transmission from a Rectangular Waveguide into Half Space through a Rectangular Aperture," Report TR-76-5, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210, May 1976.

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